Week 01: Introduction

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Lecture Overview

- Discuss Syllabus
- Course Overview
- Introduction to Dynamical Systems

Course Information

- Time
  - 9:00-noon Fridays (Lecture1 + Lecture2 + Exercise)
- Instructor
  - Prof. Selman Sakar
- Email
  - selman.sakar@epfl.ch
- Teaching Assistants (+8 student assistants)
  - Houman Javaheri: houman.javaheri@epfl.ch
  - MATLAB Assignments: Lorenzo Noseda: lorenzo.noseda@epfl.ch

Course Material

- Polycopié is on Moodle
  - Solved exercises
- Relevant External Resources
  - System Dynamics, Katsuhiko Ogata
  - System Dynamics, William Palm

- Course Webpage
  - https://moodle.epfl.ch/course/view.php?id=13902
  - Problem Sets and Solutions
  - MATLAB assignments and tutorials
  - Lecture notes
  - Exams from previous years

Grading

- Written Final Exam

- Emphasis on understanding major concepts
  - i.e. no need for calculator, memorization of formulas

- MATLAB Assignments (total of 3)
  - Pass (1 point), fail (0 point), and distinction (1.5 point)
  - Do not copy-paste reports
  - 3 or 3.5 points: 0.25 added to your final grade
  - 4 or 4.5 points: 0.5 added to your final grade

Class Goals

- Combine previous knowledge (differential equations, classical mechanics, matrix algebra) with new mathematical tools (i.e. Laplace transform, Bode plots) for the design and analysis of dynamical systems

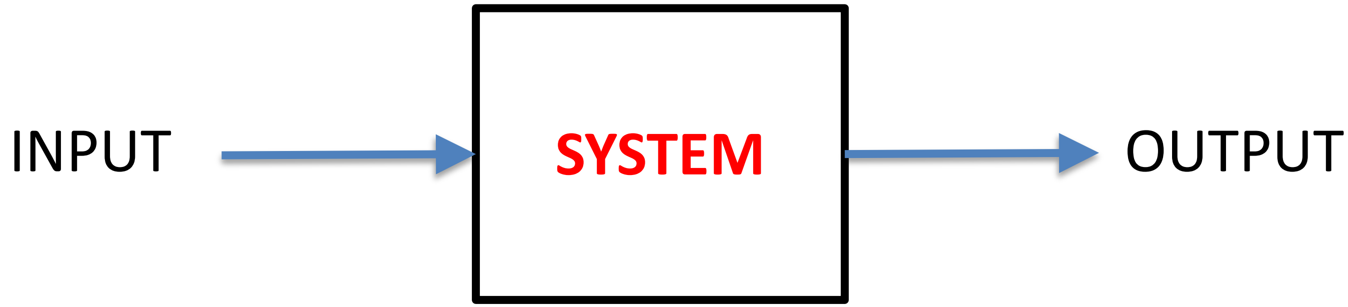
- A systems level understanding of dynamic performance

- Preparation for more advanced courses such as Control Systems, System Identification, Vibrations, Robotics

Contents

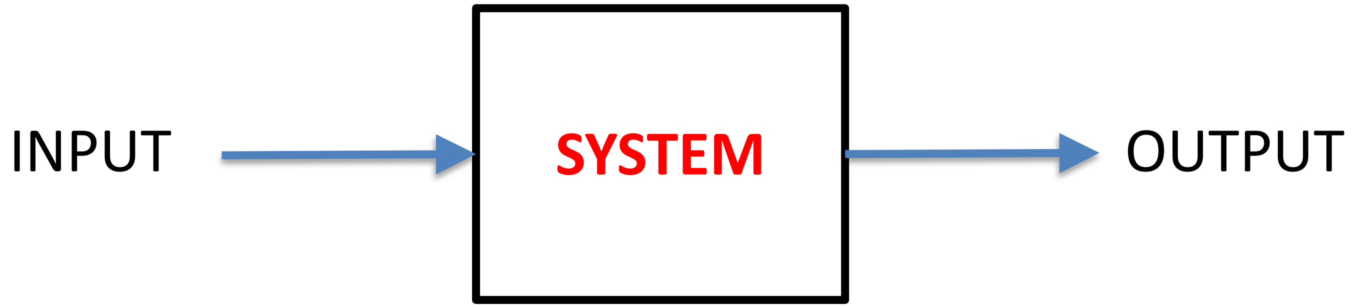
- Mathematical Modeling of Dynamical Systems
  - Mechanical, electrical, and electromechanical
  - Analogous systems
- State-space Representation of LTI Systems (Linear Algebra)
- Linearization for Nonlinear Systems (Jacobian)
- Laplace Transform and Transfer Function
- Transient Response (time domain analysis, poles and zeros)
- Frequency Response (frequency domain analysis, Bode plot, Nyquist)

System



- A combination of components acting together to perform a specific objective
- **Dynamics**: The evolution of the system over time
- Physical Systems: Machines, electronic circuits
- Non-physical Systems: Financial, software, social network
- Cyber-physical Systems: Autonomous systems, robots

Block Diagram Representation

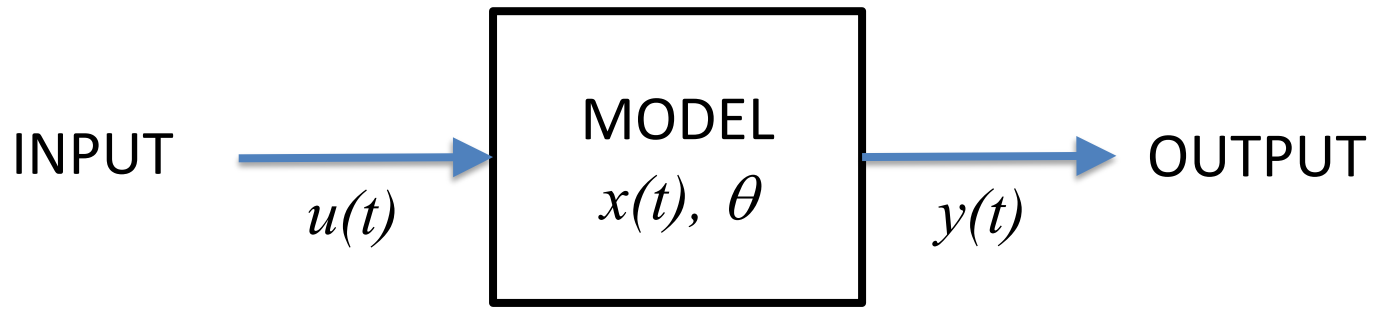


- Representation of a system in terms of discrete blocks that represent part of a system

- Arrows indicate flow of signals

- The box may include a set of differential equations that relate input to output or a Transfer Function (will come later when we learn Laplace Transform)

Mathematical Representation of a Dynamical System

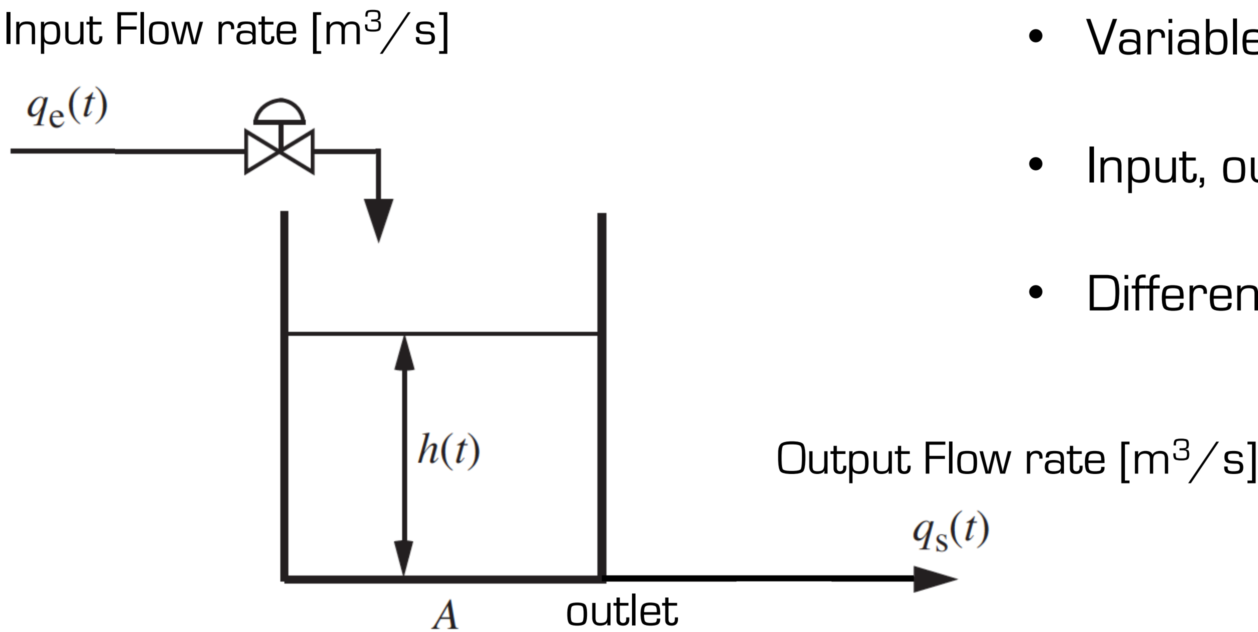


- **States of a System**: Internal variables that change in time
- **Model of a System:** States ($x(t)$) and Parameters (θ)

- Mathematical representation of a system using **differential equations**

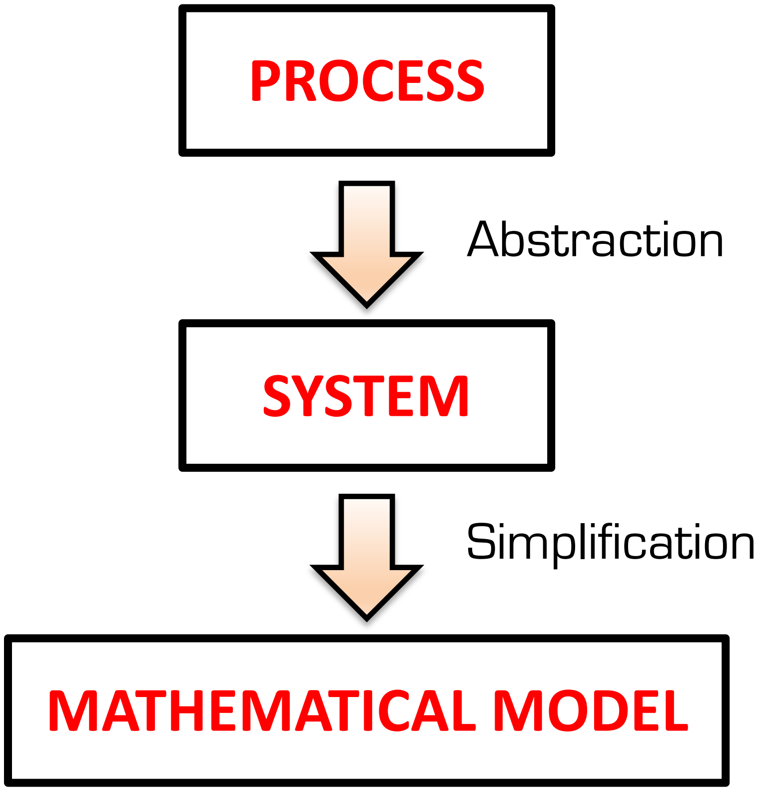
- **Mono-variable:** Single input and single output
- **Multivariable:** Multiple inputs and/or outputs

Example



- Variables and Parameters
- Input, output, and states
- Differential equations

Modeling Approach

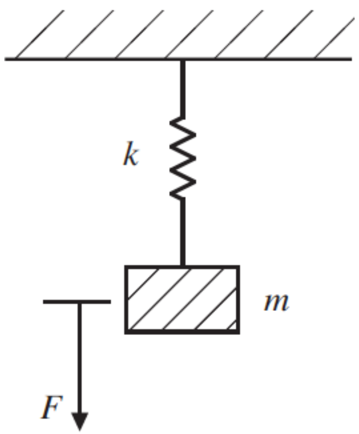


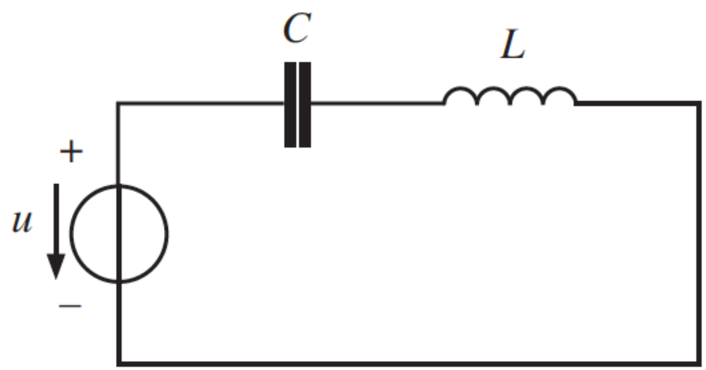
Modeling Approach

- A mathematical representation that is general enough to include all necessary physical entities
- Find basic constituent elements, study them in detail, and understand the interaction among them

- The choice of states is not unique

- **Analogous mathematical representations** of different systems





Inertia ↔ Inductance
Spring ↔ Capacitance
Friction ↔ Resistance

Case Study: The trajectory of a car

PROCESS

Abstraction

SYSTEM

PERTURBATIONS:
Wind, snow, slope

INPUT:
Gas, brake,
steering wheel

SYSTEM

OUTPUT:
Position,
velocity,
acceleration

Case Study: The trajectory of a car

PROCESS

Abstraction

SYSTEM

- **States**:
  - Engine: Valves, pistons, camshaft
  - Brake hydraulics: pressure, force
  - Steering mechanisms: transmission, motor speed, torque

Case Study: The trajectory of a car

SYSTEM

Simplification

MODEL

PERTURBATIONS:
Independent variables

INPUT:
Independent variables

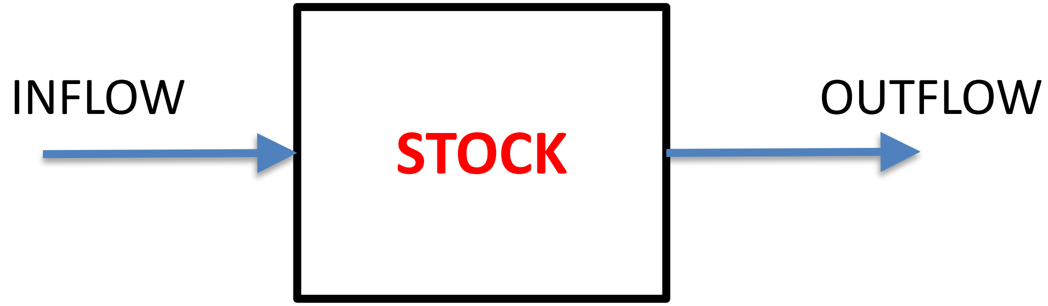
MATH MODEL

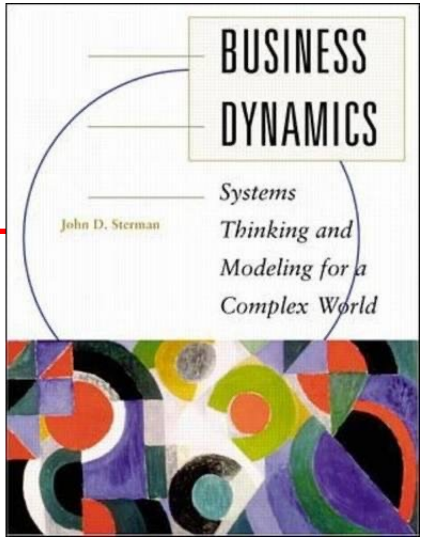
OUTPUT:
Dependent variables

Perturbations

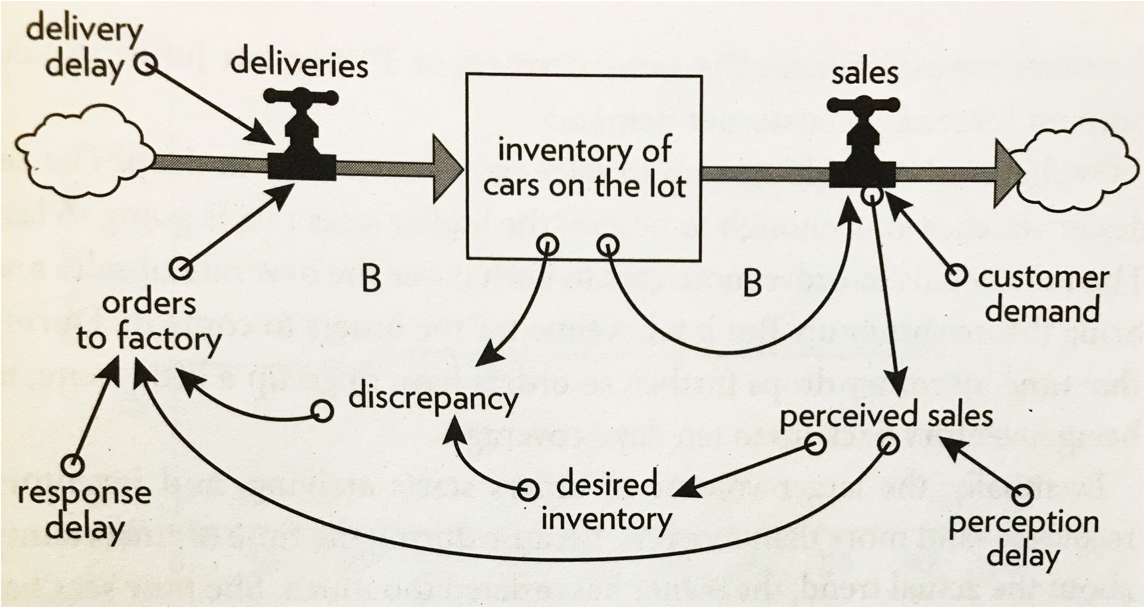
- The influence of the environment
- Undesirable, unknown, non-adjustable
- May be deterministic or stochastic in nature
- Often neglected for simplification
- Concept of robustness and resilience
- Active control for stabilization

Business Dynamics: Stock-flow Diagrams

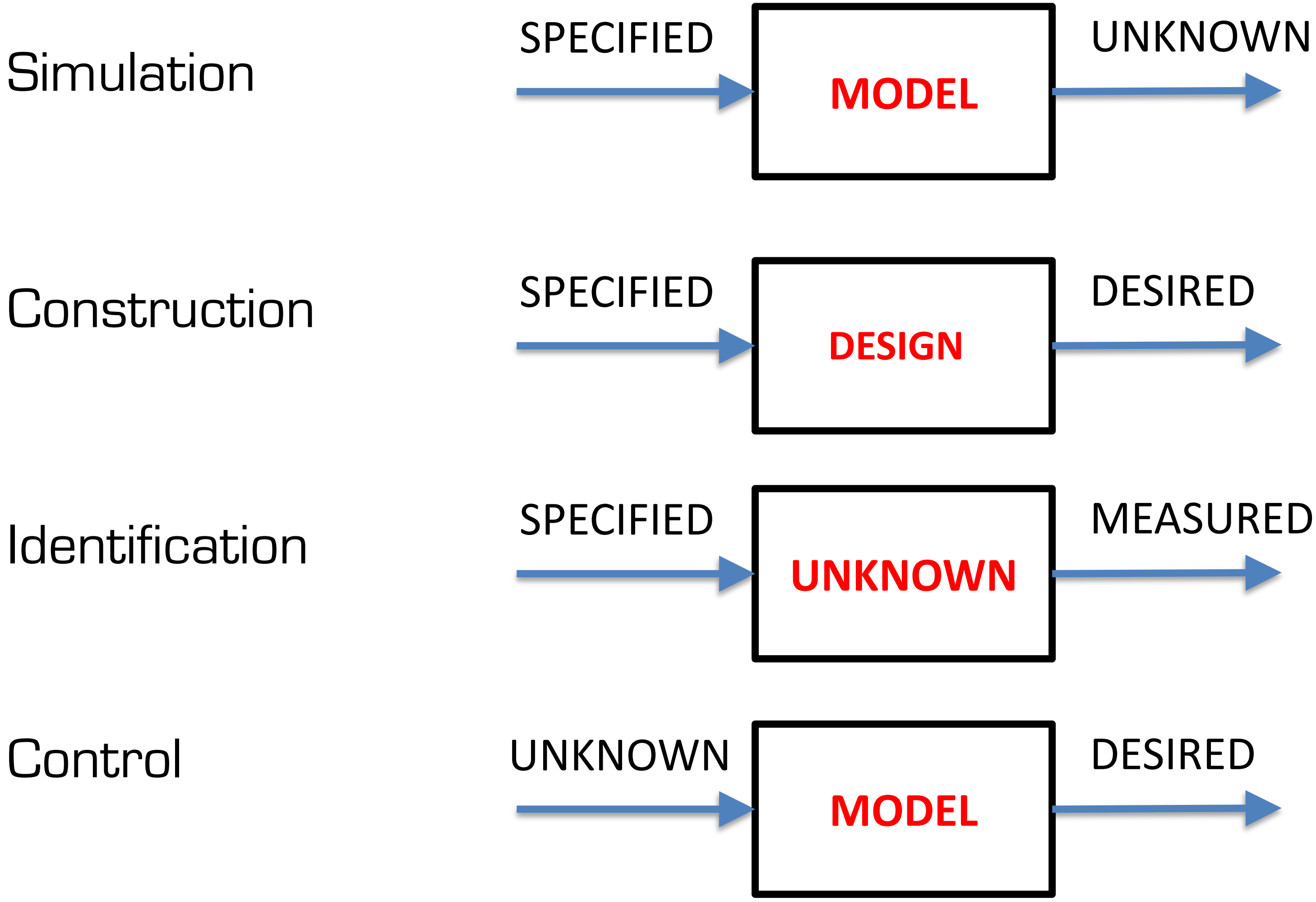




Introduction to System Dynamics
https://www.youtube.com/watch?v=AnTwZVviXyY



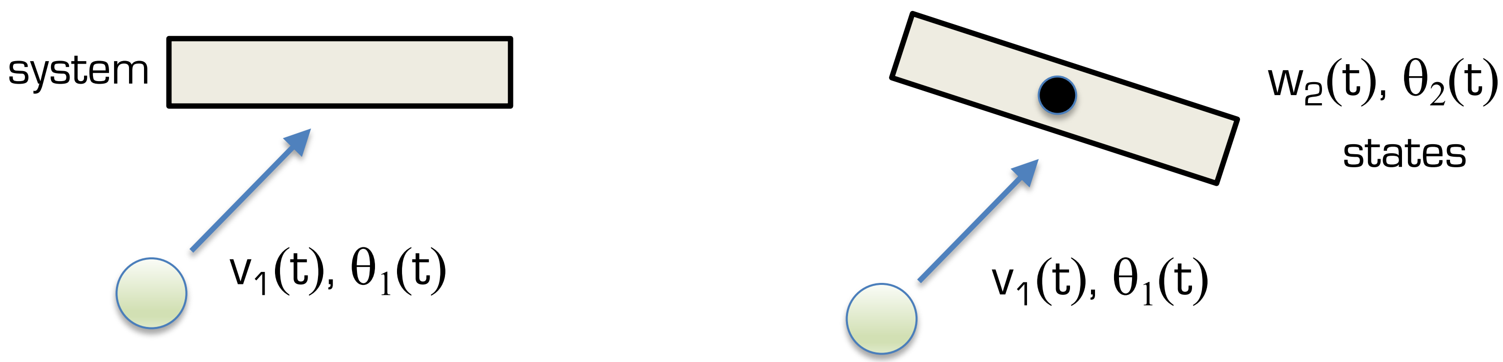
Domains of Application



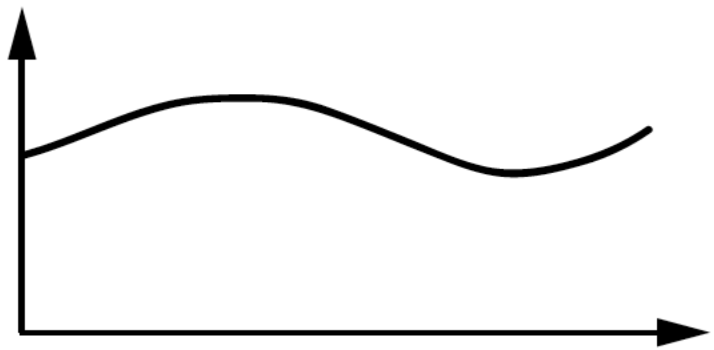
Static and Dynamical Systems

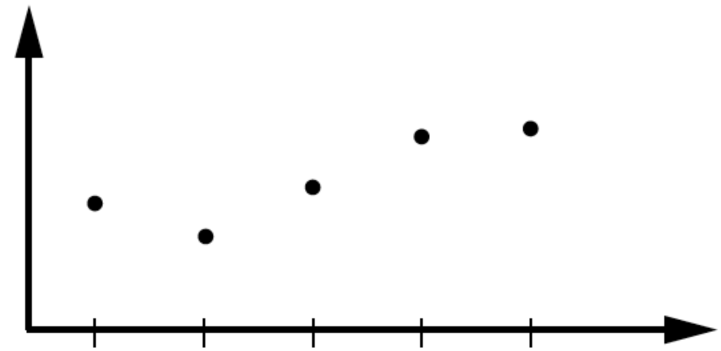
- **Static System**: All states and parameters are constant. The output only depends on the input. $y(t) = f[u(t)]$

- The state of **dynamical systems** varies over time and obeys differential equations that involve time derivatives (or difference equations if the system is discrete)
- A prediction about the system's future behavior requires
  - An analytical solution of differential equations
  - Numerical integration over time through computer simulation



Continuous and Discrete-Time Systems





- Discrete-time systems: Countable number of states (sampling time)
- Computer programs, financial models

- Difference equations

$$x(k+1) = f(x(k), u(k)) \qquad k \in \mathbb{Z}$$

Linear and Nonlinear Systems

- A system is consider linear if it obeys superposition principle, which is defined by additivity and homogeneity properties.

Additivity: $f(x + y) = f(x) + f(y)$

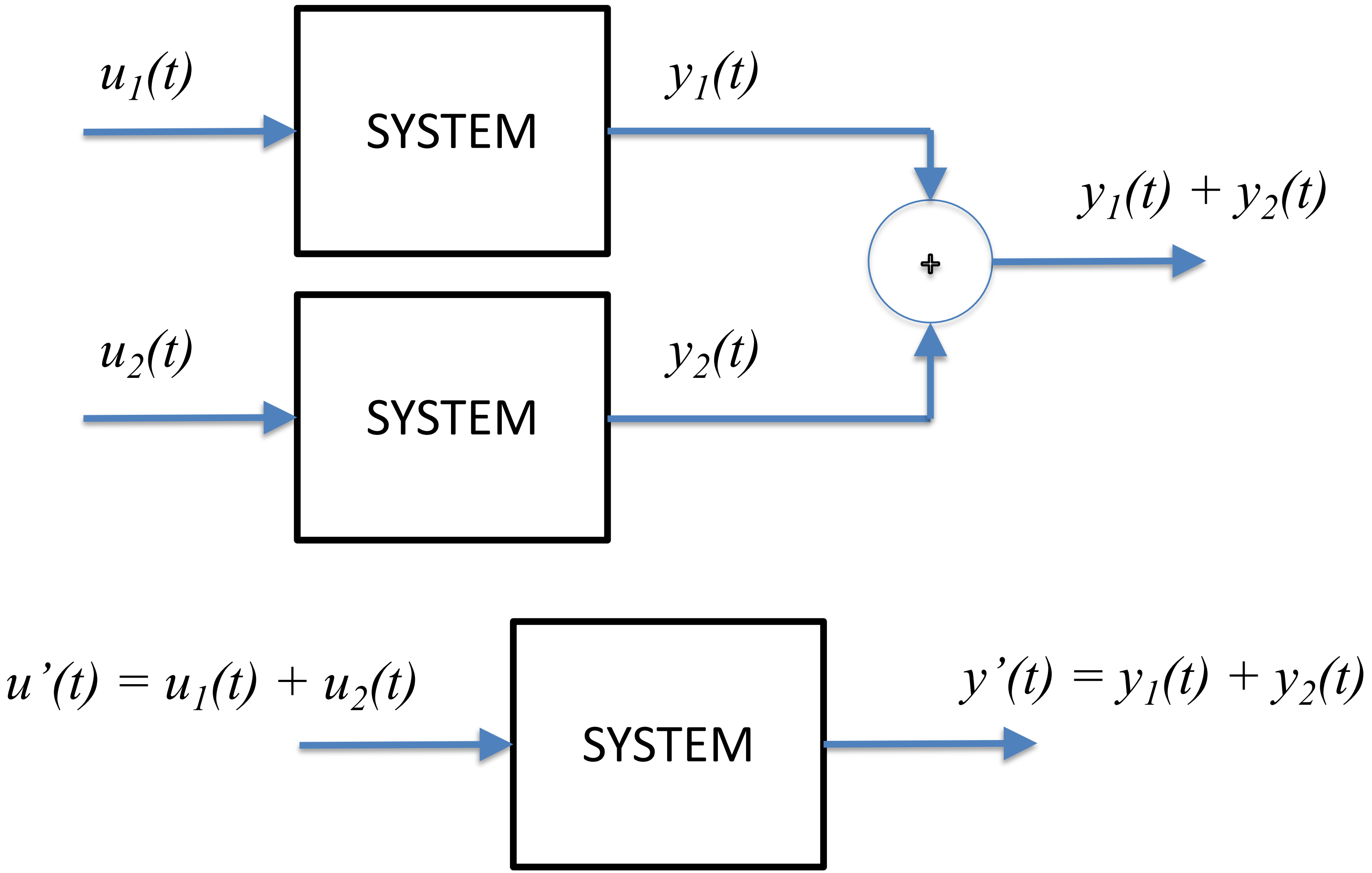
Homogeneity: $f(\alpha x) = \alpha f(x)$ for scalar α

For all linear systems, the net response caused by two or more inputs is the sum of the responses that would have been caused by each input individually

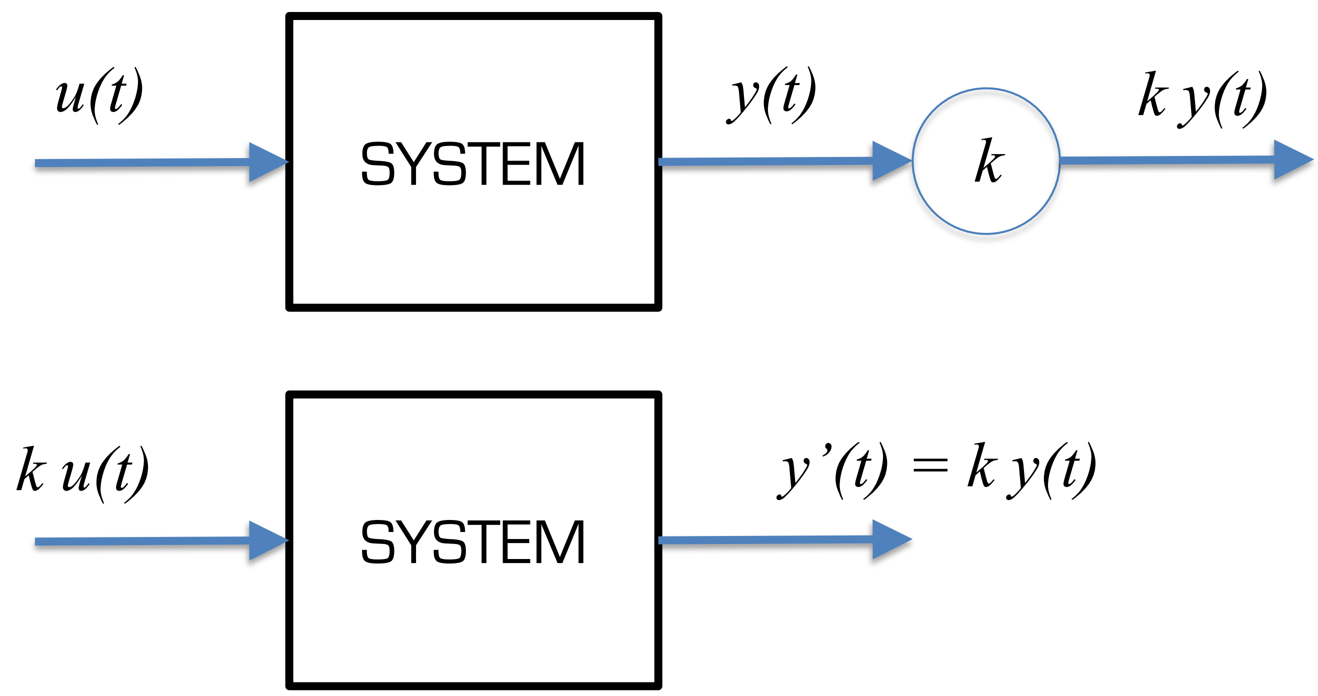
A system defined as $y(t) = h[u(t)]$ is considered linear if

$$h[\alpha u_1(t) + \beta u_2(t)] = \alpha h[u_1(t)] + \beta h[u_2(t)] = \alpha y_1(t) + \beta y_2(t)$$

Linear and Nonlinear Systems: Additivity



Linear and Nonlinear Systems: Homogeneity



Note: k is a scalar

Rule of thumbs

- System linearity is independent of scaling in time
  - $y(t) = u(\sin(t))$
  - $y(t) = u(t^2)$

- Non-linear input terms will obviously make the system nonlinear
  - $y(t) = u^2(t)$

- System linearity is independent of the coefficients (can be time-varying)
  - $y(t) = \sin(t)\, u(t)$

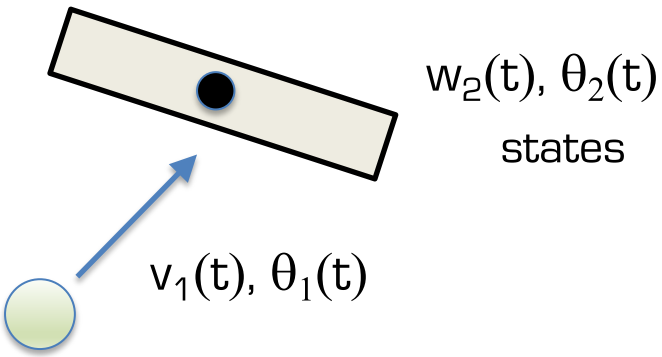
- System linearity depends on terms that do not depend on the input or output
  - $y(t) = 2t + u(t)$
  - $y(t) = 2 + u(t)$

Time Invariance

- A system is time-invariant if its parameters do not change in time.
- The physical properties of the system remain constant (i.e. mass, geometry)
- For the same process, one can choose to generate a time-invariant or time-variant model. For example, rocket and fuel tank.

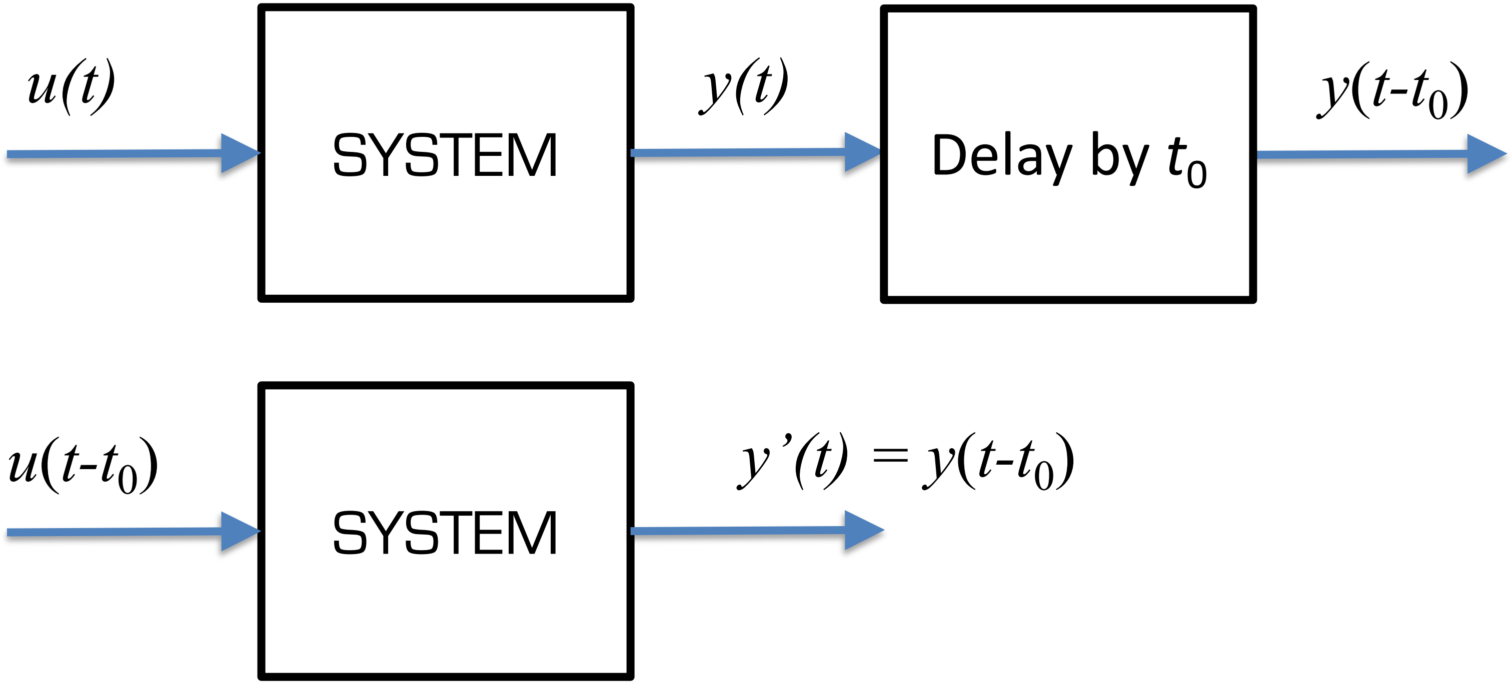
- Mass as a parameter of the system that varies with time: time-variant system
- Mass as a state variable of the system and the rest of the model parameters are constant: time-invariant system

Time Invariance

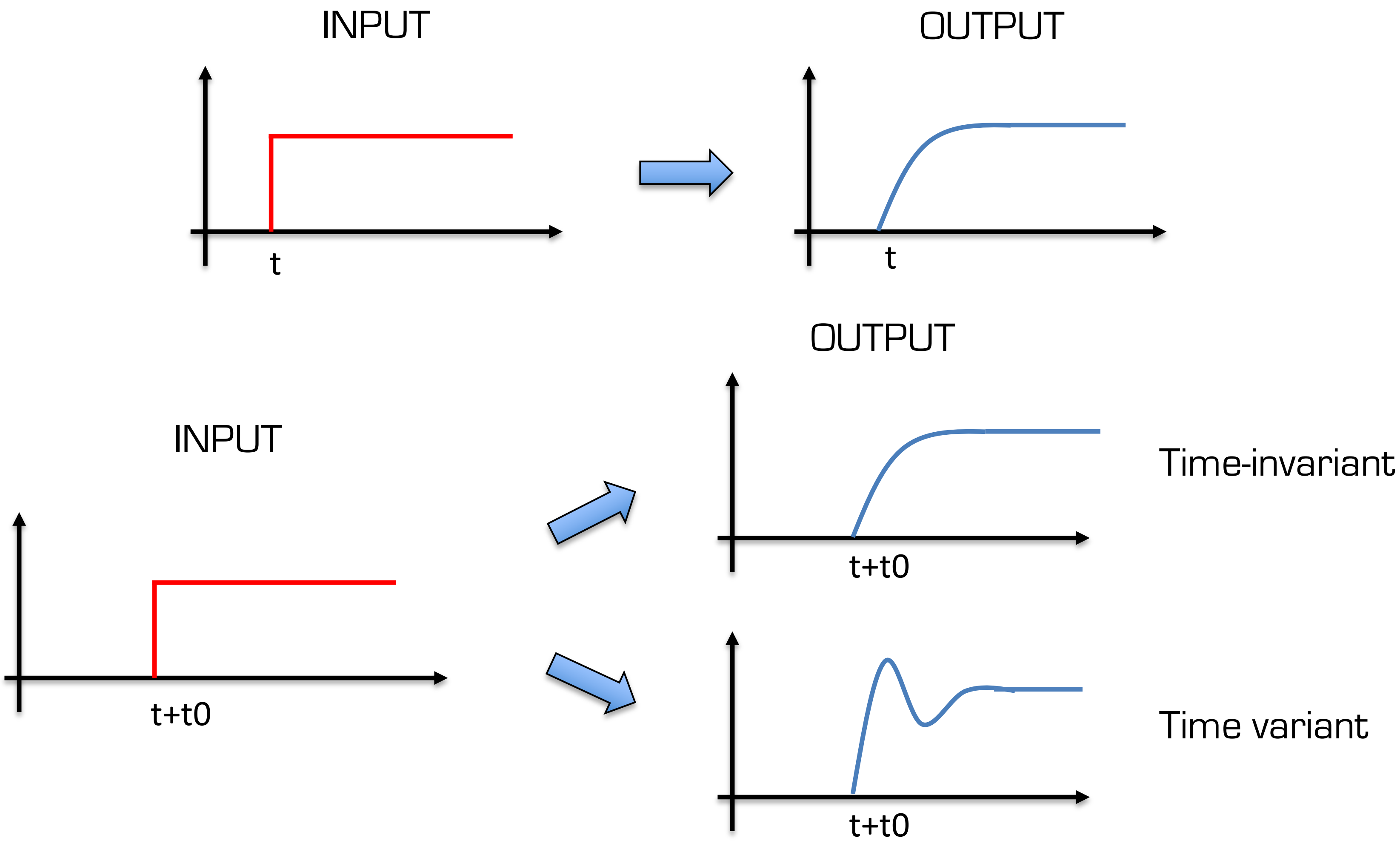


Time Invariance

- From a different perspective, the system does not age and the input does not change the foundation of the system
- Mathematically speaking, if input $u(t)$ generates $y(t)$ then the output of the system in response to the shifted input $u(t\text{-}t_0)$ is $y(t\text{-}t_0)$.



Graphical Representation



Rule of thumbs

- Time-invariance depends on scaling in time
- $y(t) = u(\sin(t))$
- $y(t) = u(t^2)$

- Non-linear input terms do not affect time-invariance
- $y(t) = u^2(t)$

- Time-invariance depends on coefficients that depend on time
- $y(t) = \sin(t)\, u(t)$

- Time-invariance depends on additional terms IF they are functions of time
- $y(t) = 2t + u(t)$
- $y(t) = 2 + u(t)$

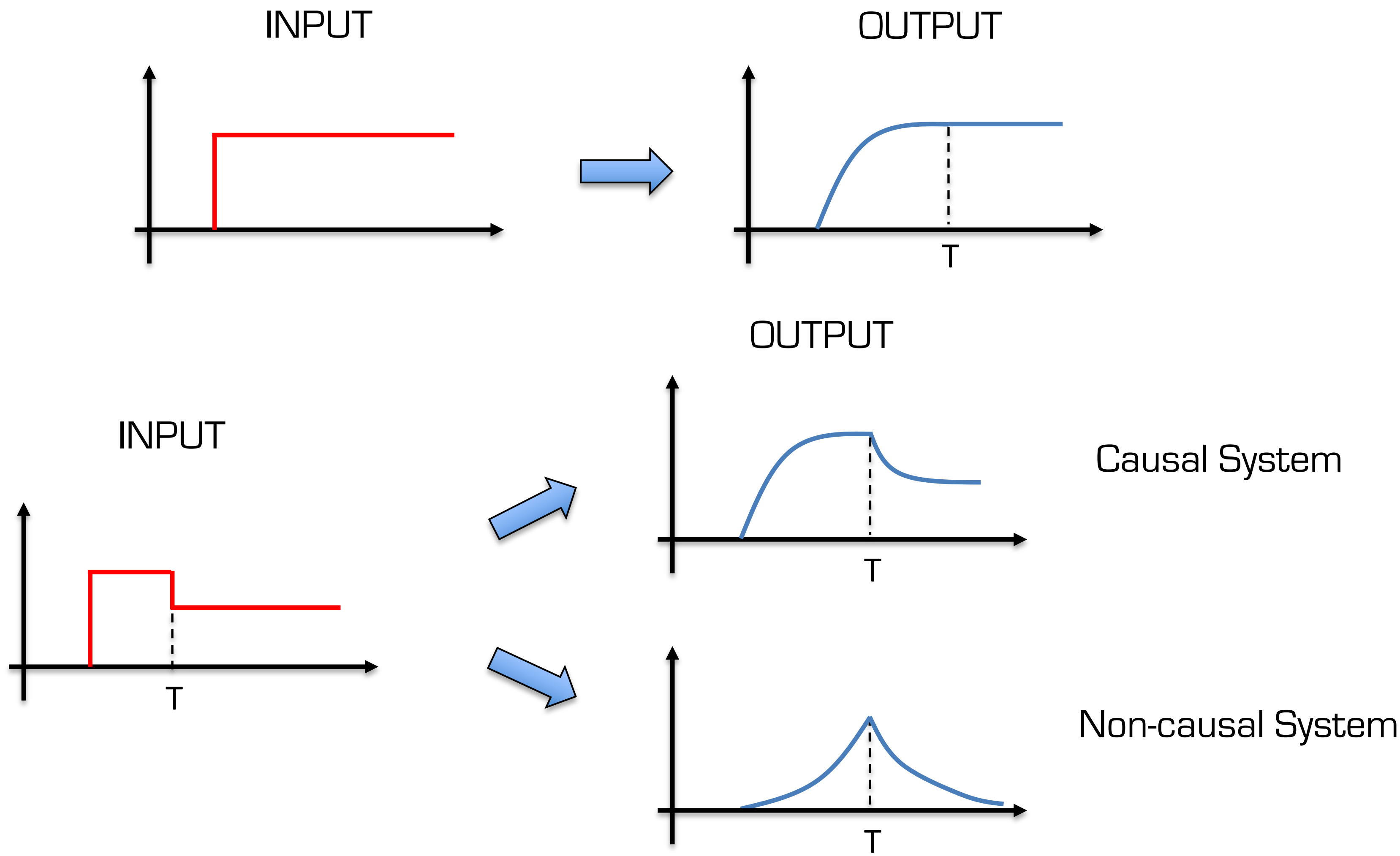
Causality

- Output depends only on past and present, but not future inputs
- An effect cannot occur before its cause

- Majority of physical systems are considered causal

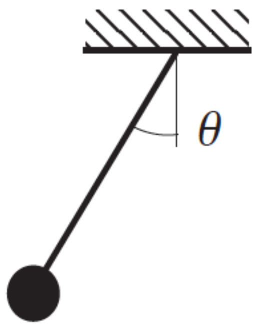
- y(t) = u(t+1) is non-causal

Graphical Representation



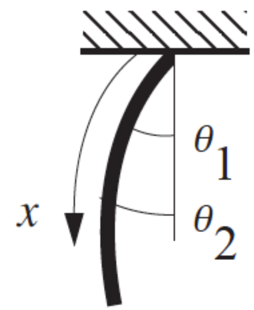
Distributed Parameters

- Dynamical systems with localized parameters is described by time-dependent ordinary differential equations.
- Dynamical systems with distributed parameters are described by partial differential equations that depend both on space and time.
- Finite Element Modeling



$$\theta = \theta(t)$$

Rigid beam with point mass



$$\theta = \theta(x, t)$$

Flexible beam with distributed mass

Stochastic Systems

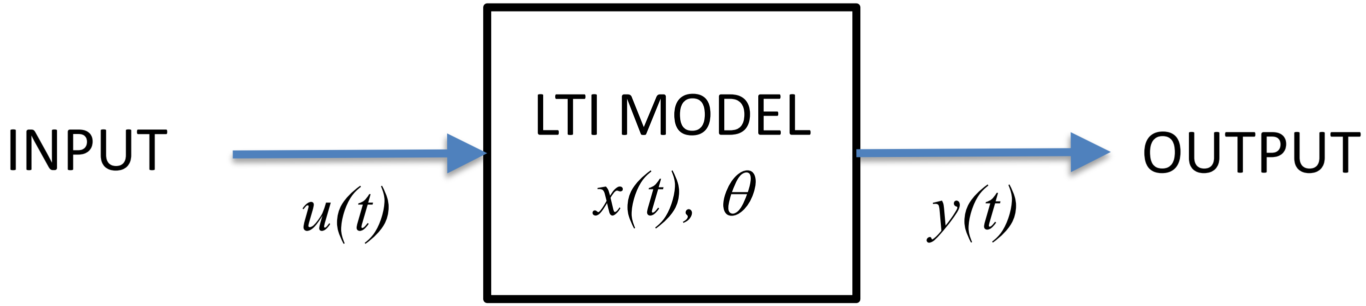
- One or more parts of the system has randomness associated with it
  - Random time delays, noisy disturbances, unpredictability of parameters

- The same set of parameter values and initial conditions will lead to an ensemble of different output values (mean, standard deviation etc.).

- Statistical mechanics, quantum mechanics, economy and finance (forecast).

Summary

- In this course we will primarily work with continuous-time, linear, time-invariant dynamical (LTI) systems.



- These systems can be represented by ordinary differential equations with constant coefficients.
- Nonlinear systems will be linearized